Compressed Sensing – a Metaphor

Mathias Blasche

Siemens Healthineers, Erlangen, Germany

Compressed Sensing (CS)[1] is an exciting new method with the potential to accelerate MR scans beyond what is possible with any other method. However, the CS reconstruction method is more complicated than the straight-forward Fourier transform used in conventional MR imaging.

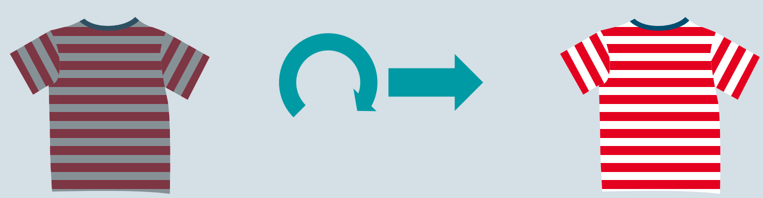
[1] 510(k) pending. Compressed Sensing Cardiac Cine is not commercially available. Future availability cannot be guaranteed.

The three key components of CS – incoherent sub-sampling, transform sparsity, and iterative reconstruction – may sound 'unwieldy' to many readers.

The 'Washing Metaphor' described below compares the CS mechanism with the washing of a T-shirt. This might look a little funny (and in fact this is intended), but it is actually quite an accurate analogy. It might help understanding CS in a 'non-technical' sort of way. Enjoy!

The goal of Compressed Sensing is to remove the (noise-like) aliasing artifacts from the image (that are due to the incoherent sub-sampling of the measured *k*-space) while ensuring that the reconstructed image is still consistent with the measured data.

1A 1B 1C



1 **Goal of Compressed Sensing**

(1A) MRI: We measure only a part of *k*-space (sub-sampling) in an incoherent ('random') way. This results in an image with noise-like aliasing artifacts.

Metaphor: We have a dirty T-shirt[1]. The dirt is homogeneously grey.

(1B) MRI: By maximizing transform sparsity[2], we remove the noise (= the aliasing artifacts) from the image. We take care that we keep the reconstructed image consistent with the measured data. The simultaneous improvement of sparsity and data consistency is done in an alternating fashion, iteratively until the optimum is achieved.

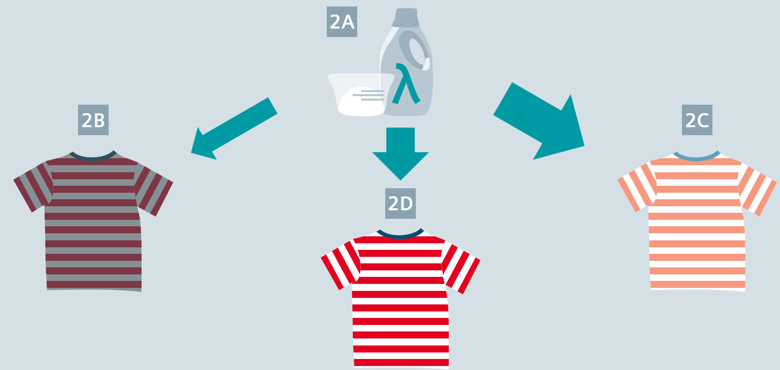
Metaphor: We wash the T-shirt in the rotating tub of the washing machine to remove the grey dirt. We take care not to wash out the colors.

(1C) MRI: The result is an anatomically correct image without aliasing artifacts (noise). The image looks virtually identical to an image with a completely measured *k*-space – but at much shorter scan time.

Metaphor:
The T-shirt is clean, and the colors have been preserved. The T-shirt looks like new.

[1] Note: The red-white striped T-shirt was chosen as an arbitrary example. The stripes have nothing to do with any 'regular' sampling of *k*-space.
[2] 'Sparsity' stands for 'transform sparsity' in the remainder of the text, i.e. the sparsity in W-space, e.g. the Wavelet domain. Increasing sparsity in W-space corresponds to decreasing noise in image space.

In order to achieve the optimal result, the sum of sparsity and data consistency is maximized. This corresponds to a minimization of noise (= aliasing artifacts) while keeping anatomical accuracy. The sparsity needs to be optimally balanced with respect to the data consistency.



2 **Sparsity vs. Data Consistency**

(2A) MRI: For optimal balancing between sparsity and data consistency, the correct weighting factor λ needs to be applied. λ is application dependent and is optimized during the application development.[3]	**Metaphor:** The right amount of detergent has to be used.
(2B) MRI: If the sparsity weighting is too low (λ too small), there is still noise (aliasing artifacts) in the image.	**Metaphor:** With too little detergent, we will not remove the dirt on the T-shirt.
(2C) MRI: If the sparsity weighting is too high (λ too large), we will lose data consistency. The image is noise free but it does not show the true anatomy.	**Metaphor:** Using too much detergent, the dirt will be gone – but also the colors.
(2D) MRI: The right balance between sparsity and data consistency (optimal λ) results in the anatomy being depicted correctly, without noise (aliasing artifacts). The image looks as if we had measured *k*-space completely – but at much shorter scan time.	**Metaphor:** With the right amount of detergent, the dirt can be removed without affecting the colors. The T-shirt looks like new.

[3] To avoid misunderstandings: λ is preset, it is optimized for the application. It is not changed in the iterative reconstruction. Rather, the iterative reconstruction improves data consistency and sparsity in an alternating fashion, based on the preset λ.

Finally, we want to emphasize again the importance of incoherent ('random') scanning for a successful CS reconstruction.

3 **The Importance of Incoherent Scanning**

(3A) MRI: In case we do not sample *k*-space 'randomly enough', the aliasing artifacts will appear in a discrete way and not noise-like. Without this critical prerequisite for CS, the reconstruction will fail[4].	**Metaphor:** If we have concentrated stains instead of 'homogeneous dirt', it will not be possible to wash the stains off without destroying the colors.
(3B) MRI: If we concentrate on data consistency, we will not get rid of the artifacts (which are not noise-like but discrete).	**Metaphor:** If we concentrate on not washing off the colors (by using a low amount of detergent), we will not succeed in removing the concentrated stains completely.
(3C) MRI: If we emphasize sparsity (complete removal of the discrete high-intensity aliasing artifacts which are not noise-like), this will go at the expense of data consistency. The anatomy is not depicted correctly.	**Metaphor:** If we use enough detergent to wash off the stains, the colors will also fade out.

[4] For the sake of simplicity, we are discussing a pure CS reconstruction without the combination with parallel imaging. The combination with parallel imaging would also address discrete aliasing artifacts to a certain degree.